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A New Bottom-Up Approach for the Construction of Fermion Mass Matrices*

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Abstract

We describe a new technique which enables one to construct an SO(10)-symmetric fermion mass matrix model at the supersymmetric grand unification scale directly from the fermion mass and mixing data at the low energy scale. Applications to two different neutrino mass and mixing scenarios are given.

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1 Description of the Technique

A new bottom-to-up approach proposed by us is summarized briefly in this talk with more details and references presented elsewhere.[1] We shall restrict our attention here to the construction of complex symmetric mass matrices arising with Higgs in the **10** and **126** representations of SO(10). The procedure allows us to construct mass matrices which exhibit as simple an SO(10) structure as possible with the maximum number of texture zeros allowed for the neutrino mass and mixing scenario in question.

1) Start from a set of quark and lepton masses and mixing matrices completely specified at the low scales.

2) Evolve the masses and mixing matrices to the GUT scale by making use of the one-loop renormalization group equations (RGEs) for the minimal supersymmetric standard model.

For this purpose we set $\Lambda_{SUSY} = 170$ GeV and $\Lambda_{GUT} = 1.2 \times 10^{16}$ GeV. Following Naculich,[2] we use the approximation that only the top and bottom quarks as well as the tau lepton contribute to the non-linear Yukawa terms in the RGEs. With a physical top mass expected to be near 160 GeV, we take the running mass to be $m_t(m_t) = 150$ GeV and adjust $m_b(m_b)$ and $\tan\beta$, so consistency is achieved at Λ_{GUT} which requires complete Yukawa unification with $\tan\beta \simeq 48.9$. We are working under the assumption that only one SO(10) **10** of Higgs contributes to the 33 elements of the up, down and charged lepton mass matrices.

3) Construct a numerical set of complex-symmetric mass matrices M^U, M^D, M^E and $M^{N_{eff}} = M^{N_{Dirac}} M_R^{-1} M^{N_{Dirac}T}$ for the up and down quarks, charged leptons and light neutrinos by making use of a procedure due to Kusenko,[3] now applied to both quarks and leptons.

Since the quark CKM mixing matrix is unitary and represents an element of the unitary group U(3), one can express it in terms of one Hermitian generator of the corresponding U(3) Lie algebra times a phase parameter α by writing

$$V_{CKM} = U'_L U_L^\dagger = \exp(i\alpha H) \tag{1a}$$

where

$$i\alpha H = \sum_{k=1}^{3} (\log v_k) \frac{\prod_{i\neq k}(V_{CKM} - v_i I)}{\prod_{j\neq k}(v_k - v_j)} \tag{1b}$$

in terms of the eigenvalues v_j of V_{CKM}, by making use of Sylvester's theorem.[3] The transformation matrices from the weak to the mass bases are given in terms of the same generator but modified phase parameters such that

$$U'_L = \exp(i\alpha H x_q), \qquad U_L = \exp\left[i\alpha H(x_q - 1)\right] \tag{1c}$$

and relation (1a) is preserved. The complex symmetric quark mass matrices in the weak basis are then related to those in the diagonal mass basis by

$$M^U = U_L'^\dagger D^U U_L'^{\dagger T}, \qquad M^D = U_L^\dagger D^D U_L^{\dagger T} \tag{1d}$$

It suffices to expand V_{CKM}, U'_L and U_L to third order in α in order to obtain accurate numerical results for the mass matrices M^U and M^D. Similar expressions can be obtained for the light neutrino and charged lepton mass matrices from the lepton mixing matrix and its eigenvalues with x_ℓ replacing x_q.

The parameters x_q and x_ℓ control the choice of bases for the quark and lepton mass matrices, respectively. The up quark mass matrix is diagonal for $x_q = 0$, while the down quark mass matrix is diagonal for $x_q = 1$; likewise, the light neutrino mass matrix is diagonal for $x_\ell = 0$, while the charged lepton mass matrix is diagonal for $x_\ell = 1$.

4) Vary x_q and x_ℓ and the signs of the mass eigenvalues to search for simplicity in the SO(10) framework, i.e., pure **10** or pure **126** contributions for as many mass matrix elements as possible.

For this purpose we note that **10**'s contribute equally to the down quark and charged lepton mass matrices, while **126** contributions differ by a factor of -3; likewise for the up quark and Dirac neutrino mass matrices. In terms of the Yukawa couplings and the appropriate VEVs, the mass matrices are given by

$$\begin{aligned} M^U &= \sum_i f^{(10_i)} v_{ui} + \sum_j f^{(126_j)} w_{uj} \\ M^D &= \sum_i f^{(10_i)} v_{di} + \sum_j f^{(126_j)} w_{dj} \\ M^{N_{Dirac}} &= \sum_i f^{(10_i)} v_{ui} - 3\sum_j f^{(126_j)} w_{uj} \\ M^E &= \sum_i f^{(10_i)} v_{di} - 3\sum_j f^{(126_j)} w_{dj} \end{aligned} \tag{2}$$

5) For the best choice of x_q and x_ℓ which maximizes the simplicity, construct a simple model of the mass matrices with as many texture zeros as possible.

6) Evolve the mass eigenvalues and mixing matrices determined from the model at the SUSY GUT scale to the low scales and compare the results with the starting input data.

2 Application to Two Different Neutrino Mass and Mixing Scenarios

We now illustrate the technique by applying it to two different neutrino scenarios, both of which explain the solar neutrino depletion data with the non-adiabatic Mikheyev-Smirnov-Wolfenstein[4] (MSW) effect, where one includes the atmospheric neutrino depletion phenomenon[5] while the other accepts the cocktail model[6] interpretation of missing dark matter.

We take the same quark input data for both models. For the light quark masses, we shall adopt the values quoted by Gasser and Leutwyler[7] while the heavy quark masses are

specified at their running mass scales:

$$
\begin{aligned}
m_u(1\text{GeV}) &= 5.1 \text{ MeV}, & m_d(1\text{GeV}) &= 8.9 \text{ MeV} \\
m_c(m_c) &= 1.27 \text{ GeV}, & m_s(1\text{GeV}) &= 175 \text{ MeV} \\
m_t(m_t) &= 150 \text{ GeV}, & m_b(m_b) &\simeq 4.25 \text{ GeV}
\end{aligned}
\tag{3a}
$$

For the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix, we adopt the following central values at the weak scale

$$
V_{CKM} = \begin{pmatrix} 0.9753 & 0.2210 & (-0.283 - 0.126i) \times 10^{-2} \\ -0.2206 & 0.9744 & 0.0430 \\ 0.0112 - 0.0012i & -0.0412 - 0.0003i & 0.9991 \end{pmatrix} \tag{3b}
$$

where we have assumed a value of 0.043 for V_{cb} and applied strict unitarity to determine V_{ub}, V_{td} and V_{ts}.

2.1 Lepton Scenario (A) involving the Non-adiabatic MSW Solar and Atmospheric Neutrino Depletion Effects

In this scenario, we single out the central points in the two neutrino mixing planes

$$
\begin{aligned}
\delta m_{12}^2 &= 5 \times 10^{-6} \text{ eV}^2, & \sin^2 2\theta_{12} &= 8 \times 10^{-3} \\
\delta m_{23}^2 &= 2 \times 10^{-2} \text{ eV}^2, & \sin^2 2\theta_{23} &= 0.5
\end{aligned}
\tag{4}
$$

With the neutrino masses assumed to be non-degenerate, we take for the lepton input

$$
\begin{aligned}
m_{\nu_e} &= 0.5 \times 10^{-6} \text{ eV}, & m_e &= 0.511 \text{ MeV} \\
m_{\nu_\mu} &= 0.224 \times 10^{-2} \text{ eV}, & m_\mu &= 105.3 \text{ MeV} \\
m_{\nu_\tau} &= 0.141 \text{ eV}, & m_\tau &= 1.777 \text{ GeV}
\end{aligned}
\tag{5a}
$$

and

$$
V_{LEPT}^{(A)} = \begin{pmatrix} 0.9990 & 0.0447 & (-0.690 - 0.310i) \times 10^{-2} \\ -0.0381 - 0.0010i & 0.9233 & 0.3821 \\ 0.0223 - 0.0030i & -0.3814 & 0.9241 \end{pmatrix} \tag{5b}
$$

where we have simply assumed a value for the electron-neutrino mass to which our analysis is not very sensitive and constructed the lepton mixing matrix by making use of the unitarity conditions with the same phase in (5b) as in (3b).

We evolve the masses and mixing matrices to the SUSY GUT scale and use the extended Kusenko[3] procedure to construct the mass matrices numerically. The simplest SO(10) structure for the mass matrices is found with $x_q = 0$ and $x_\ell = 0.88$, in which case the matrices have the following SO(10) transformation properties:

$$M^U \sim M^{N_{Dirac}} \sim diag(126;\ 126;\ 10) \tag{6a}$$

$$M^D \sim M^E \sim \begin{pmatrix} 10', 126 & 10', 126' & 10' \\ 10', 126' & 126 & 10' \\ 10' & 10' & 10 \end{pmatrix} \tag{6b}$$

Note that the same **10** is assumed to contribute to the 33 elements of the above mass matrices, with Yukawa coupling unification achieved for $\tan\beta \simeq 48.9$.

In this scenario we are able to construct a simple SO(10) model with just nine independent parameters for the following four matrices, such that

$$\begin{aligned} M^U &= diag(F',\ E',\ C') \qquad & M^{N_{Dirac}} &= diag(-3F',\ -3E',\ C') \\ M^D &= \begin{pmatrix} 0 & A & D \\ A & E & B \\ D & B & C \end{pmatrix} \qquad & M^E &= \begin{pmatrix} F & 0 & D \\ 0 & -3E & B \\ D & B & C \end{pmatrix} \end{aligned} \tag{7a}$$

where only D is complex and

$$C'/C = v_u/v_d, \qquad E'/E = -4F'/F = w_u/w_d \tag{7b}$$

in terms of the ratios of the **10** and of the **126** vacuum expectation values associated with the diagonal Yukawa couplings. In this model for scenario (A), two **10**'s and two **126**'s are required as indicated in (6a,b), while four texture zeros appear in the two matrices for M^U and M^D and for $M^{N_{Dirac}}$ and M^E.

With

$$F' = -1.098 \times 10^{-3}, \qquad E' = 0.314, \qquad C' = 120.3 \tag{8a}$$

$$\begin{aligned}
C &= 2.4607, && \text{so} \quad v_u/v_d = \tan\beta = 48.9 \\
E &= -0.3830 \times 10^{-1}, && \text{hence} \quad w_u/w_d = -8.20 \\
F &= -0.5357 \times 10^{-3}, \quad & B &= 0.8500 \times 10^{-1} \\
A &= -0.9700 \times 10^{-2}, \quad & D &= (0.4200 + 0.4285i) \times 10^{-2}
\end{aligned} \tag{8b}$$

in GeV, the masses and mixing matrices are calculated at the GUT scale by use of the projection operator technique of Jarlskog[8] and then evolved to the low scales. The following low-scale results emerge for the quarks:

$$\begin{aligned}
m_u(1\text{GeV}) &= 5.10 \text{ MeV}, & m_d(1\text{GeV}) &= 9.33 \text{ MeV} \\
m_c(m_c) &= 1.27 \text{ GeV}, & m_s(1\text{GeV}) &= 181 \text{ MeV} \\
m_t(m_t) &= 150 \text{ GeV}, & m_b(m_b) &= 4.09 \text{ GeV}
\end{aligned} \tag{9a}$$

$$V_{CKM} = \begin{pmatrix} 0.9753 & 0.2210 & (0.2089 - 0.2242i) \times 10^{-2} \\ -0.2209 & 0.9747 & 0.0444 \\ 0.0078 - 0.0022i & -0.0438 - 0.0005i & 0.9994 \end{pmatrix} \tag{9b}$$

which are to be compared with the input starting data given in (3a) and (3b).

In the absence of any VEVs coupling the left-handed neutrino fields together, we observe that the heavy righthanded Majorana neutrino mass matrix can be computed at the GUT scale from the seesaw mass formula

$$M^R = -M^{N_{Dirac}}(M^{N_{eff}})^{-1}M^{N_{Dirac}} \tag{10a}$$

which can be well approximated by the nearly geometric form

$$M^R = \begin{pmatrix} F'' & -\frac{2}{3}\sqrt{F''E''} & -\frac{1}{3}\sqrt{F''C''}e^{i\phi_{D''}} \\ -\frac{2}{3}\sqrt{F''E''} & E'' & -\frac{2}{3}\sqrt{E''C''}e^{i\phi_{B''}} \\ -\frac{1}{3}\sqrt{F''C''}e^{i\phi_{D''}} & -\frac{2}{3}\sqrt{E''C''}e^{i\phi_{B''}} & C'' \end{pmatrix} \tag{10b}$$

where $E'' = \frac{2}{3}\sqrt{F''C''}$ and $\phi_{B''} = -\phi_{D''}/3$. In terms of the three additional parameters $C'' = 0.6077 \times 10^{15}$, $F'' = 0.1745 \times 10^{10}$ and $\phi_{D''} = 45^o$, the resulting heavy Majorana neutrino masses are determined to be

$$\begin{aligned} M_{R_1} &= 0.249 \times 10^9 \text{ GeV} \\ M_{R_2} &= 0.451 \times 10^{12} \text{ GeV} \\ M_{R_3} &= 0.608 \times 10^{15} \text{ GeV} \end{aligned} \tag{10c}$$

From the model parameters, the seesaw formula and Jarlskog's projection operator technique,[8] the light lepton masses and their mixing matrix can be constructed at the GUT scale and then evolved downward to the low scales where we find

$$\begin{aligned} m_{\nu_e} &= 0.534 \times 10^{-5} \text{ eV}, \qquad & m_e &= 0.504 \text{ MeV} \\ m_{\nu_\mu} &= 0.181 \times 10^{-2} \text{ eV}, \qquad & m_\mu &= 105.2 \text{ MeV} \\ m_{\nu_\tau} &= 0.135 \text{ eV}, \qquad & m_\tau &= 1.777 \text{ GeV} \end{aligned} \tag{11a}$$

and

$$V_{LEPT} = \begin{pmatrix} 0.9990 & 0.0451 & (-0.029 - 0.227i) \times 10^{-2} \\ -0.0422 & 0.9361 & 0.3803 \\ 0.0174 - 0.0024i & -0.3799 - 0.0001i & 0.9371 \end{pmatrix} \tag{11b}$$

The agreement with the initial input values in (5a,b) is excellent.

2.2 Lepton Scenario (B) involving the Non-adiabatic MSW Solar Depletion Effect and the Cocktail Model for Mixed Dark Matter

In this scenario, the parameters of the 23 mixing plane given in (4) are replaced by

$$\delta m_{23}^2 = 49 \text{ eV}^2, \qquad \sin^2 2\theta_{23} = 10^{-3} \tag{12a}$$

with the tau-neutrino now assumed to account for the 30% hot dark matter component of the cocktail model[6] for mixed dark matter with a mass of $m_{\nu_\tau} = 7.0$ eV. The mixing angle

has been set close to the present upper bound from accelerator experiments, so the mixing matrix is now given by

$$V_{LEPT}^{(B)} = \begin{pmatrix} 0.9990 & 0.0447 & (-0.289 - 0.129i) \times 10^{-2} \\ -0.0446 & 0.9989 & 0.0158 \\ 0.0036 - 0.0013i & -0.0157 & 0.9998 \end{pmatrix} \tag{12b}$$

Following the same general procedure as applied for scenario (A), we find the simplest SO(10) structure is obtained for $x_q = 0.5$ and $x_\ell = 0.0$, but only one texture zero is then present. Instead we make use of the bases where $x_q = 0$ and $x_\ell = 0.3$ as in scenario (A). The quark mass matrices are then exactly the same as before, while the SO(10) structures are more complicated as seen from

$$M^U \sim M^{N_{Dirac}} \sim diag(10, 126;\ 126;\ 10) \tag{13a}$$

$$M^D \sim M^E \sim \begin{pmatrix} 10, 10', 126 & 10', 126' & 10' \\ 10', 126' & 126 & 10', 126' \\ 10' & 10', 126' & 10 \end{pmatrix} \tag{13b}$$

Without repeating all the details, we simply write down the form of the model matrices derived in this scenario and find

$$\begin{aligned} M^U &= diag(F',\ E',\ C') & \qquad M^{N_{Dirac}} &= diag(-2.5F',\ -3E',\ C') \\ M^D &= \begin{pmatrix} 0 & A & D \\ A & E & B \\ D & B & C \end{pmatrix} & \qquad M^E &= \begin{pmatrix} \frac{5}{6}F & -\frac{1}{3}A & D \\ -\frac{1}{3}A & -3E & \frac{1}{3}B \\ D & \frac{1}{3}B & C \end{pmatrix} \end{aligned} \tag{14}$$

where again only D is complex. In this model for scenario (B), two **10**'s and two **126**'s are required as indicated in (13a,b), while four texture zeros appear in the two matrices for M^U and M^D but only three texture zeros appear for $M^{N_{Dirac}}$ and M^E. Since the quark mass matrices must lead to the same numerical results as in scenario (A), the values for the parameters introduced above are those given in (8a) and (8b).

The heavy right-handed Majorana mass matrix can be approximated by the nearly geometric form

$$M^R = \begin{pmatrix} F'' & -2\sqrt{F''E''} & -\sqrt{F''C''}e^{i\phi_{D''}} \\ -2\sqrt{F''E''} & -E'' & 2\sqrt{E''C''}e^{i\phi_{B''}} \\ -\sqrt{F''C''}e^{i\phi_{D''}} & 2\sqrt{E''C''}e^{i\phi_{B''}} & C'' \end{pmatrix} \quad (15a)$$

where $E'' = \frac{1}{8}\sqrt{F''C''}$, aside from an overall sign. With $C'' = 0.2323 \times 10^{14}$, $F'' = 0.1096 \times 10^{11}$ and $\phi_{D''} = 18.7^o$, $\phi_{B''} = 23.0^o$ and $\phi_{C''} = 41.8^o$, we find the resulting heavy Majorana neutrino masses for this case are determined to be

$$\begin{aligned} M_{R_1} &= 0.841 \times 10^9 \text{ GeV} \\ M_{R_2} &= 0.312 \times 10^{12} \text{ GeV} \\ M_{R_3} &= 0.235 \times 10^{14} \text{ GeV} \end{aligned} \quad (15b)$$

By again making use of the simplified matrices at the GUT scale first to compute the lepton masses and mixing matrix V_{LEPT} by the projection operator technique of Jarlskog and then to evolve the results to the low scales, we find at the low scales for the (B) scenario

$$\begin{aligned} m_{\nu_e} &= 0.544 \times 10^{-6} \text{ eV}, & m_e &= 0.511 \text{ MeV} \\ m_{\nu_\mu} &= 0.242 \times 10^{-2} \text{ eV}, & m_\mu &= 107.9 \text{ MeV} \\ m_{\nu_\tau} &= 6.99 \text{ eV}, & m_\tau &= 1.776 \text{ GeV} \end{aligned} \quad (16a)$$

and

$$V_{LEPT} = \begin{pmatrix} 0.9992 & 0.0410 & (0.150 - 0.107i) \times 10^{-2} \\ -0.0411 & 0.9991 & 0.0113 \\ -0.0010 - 0.0011i & -0.0123 & 0.9999 \end{pmatrix} \quad (16b)$$

to be compared with the initial low scale input in (5a) and (12).

3 Summary

In this talk we have sketched a procedure which enables one to construct fermion mass matrices at the GUT scale which yield the low energy data taken as input. The models

constructed for the two neutrino scenarios work well in the SO(10) SUSY GUT framework with relatively few parameters, but the structure exhibited for scenario (B) with a 7 eV tau-neutrino is not as simple as that for scenario (A) based on the observed muon-neutrino atmospheric depletion effect. Discrete family symmetries giving rise to these models are now under investigation. Our general procedure to construct mass matrices can be applied to other symmetry-based frameworks as well.

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